# User-Specified General-to-Specific and Indicator Saturation Methods

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Abstract General-to-Specific (GETS) modelling provides a comprehensive, systematic and cumulative approach to modelling that is ideally suited for conditional forecasting and counterfactual analysis, whereas Indicator Saturation (ISAT) is a powerful and flexible approach to the detection and estimation of structural breaks (e.g. changes in parameters), and to the detection of outliers. To these ends, multipath backwards elimination, single and multiple hypothesis tests on the coefficients, diagnostics tests and goodness-of-fit measures are combined to produce a parsimonious final model. In many situations a specific model or estimator is needed, a specific set of diagnostics tests may be required, or a specific fit criterion is preferred. In these situations, if the combination of estimator/model, diagnostics tests and fit criterion is not offered by publicly available software, then the implementation of user-specified GETS and ISAT methods puts a large programming-burden on the user. Generic functions and procedures that facilitate the implementation of user-specified GETS and ISAT methods for specific problems can therefore be of great benefit. The R package gets, version 0.20 (September 2019), is the first software – both inside and outside the R universe – to provide a complete set of facilities for user-specified GETS and ISAT methods: User-specified model/estimator, user-specified diagnostics and user-specified goodness-of-fit criteria. The aim of this article is to illustrate how user-specified GETS and ISAT methods can be implemented.

### Introduction

General-to-Specific (GETS) modelling provides a comprehensive, systematic and cumulative approach to modelling that is ideally suited for scenario analysis, e.g. conditional forecasting and counterfactual analysis. To this end, well-known ingredients (tests off coefficients, multi-path backwards elimination, diagnostics tests and fit criteria) are combined to produce a parsimonious final model that passes the chosen diagnostics. GETS modelling originated at the London School of Economics (LSE) during the 1960s, and gained widespread acceptance and usage in economics during the 1980s and 1990s. The two-volume article collection by Campos et al. (2005) provides a comprehensive historical overview of key-developments in GETS modelling. Software-wise, a milestone was reached in 1999, when the data-mining experiment of Lovell (1983) was re-visited by Hoover and Perez (1999). They showed that automated GETS modelling could improve substantially upon the then prevalent modelling approaches. The study spurred numerous new studies and developments, including Indicator Saturation (ISAT) methods, see Hendry et al. (2008) and Castle et al. (2015). ISAT methods provide a powerful and flexible approach to the detection and estimation of structural breaks (e.g. changes in parameters), and to the detection of outliers.

On CRAN, there are two packages that provide GETS methods. The second, named **gets**, is simply the successor of the first, which is named **AutoSEARCH**. Since October 2014 the development of **AutoSEARCH** is frozen, and all development efforts have been directed towards **gets** together with Dr. Felix Pretis and Dr. James Reade. An introduction to the **gets** package is provided by Pretis et al. (2018). However, it does does not cover the user-specification capabilities of the package, some of which were not available at the time.

At the time of writing (September 2019), the publicly available softwares that provide GETS and ISAT methods are contained in Table 1. Although they offer GETS and ISAT methods for some of the most popular models in applications, in many situations a specific model or estimator will be needed, a specific set of diagnostics tests may be required, or a specific fit criterion is preferred. In these situations, if the combination of estimator/model, diagnostics tests and fit criterion is not offered by the publicly available softwares, then the implementation of user-specified GETS and ISAT methods puts a large programming-burden on the user. Generic functions and procedures that facilitate the implementation of user-specified GETS and ISAT methods for specific problems can therefore be of great benefit. The R package gets, version 0.20 (September 2019), is the first software – both inside and outside the R universe – to provide a complete set of facilities for user-specified GETS and ISAT methods: User-specified model/estimator, user-specified diagnostics and user-specified goodness-of-fit criteria. The aim of this article is to illustrate how user-specified GETS and ISAT methods can be implemented.

<sup>&</sup>lt;sup>1</sup>Both packages were created by me. Originally, I simply wanted to rename the first to the name of the second. This, however, is inconvenient in practice I was told, so I was instead asked by CRAN to publish a "new" package with the new name.

	HP1999 (MATLAB)	Autometrics (OxMetrics)	Grocer (Scilab)	gets (R)
More than 10 paths		Yes	Yes	Yes
GETS of linear regression	Yes	Yes	Yes	Yes
GETS of variance models (log-ARCH)				Yes
GETS of logit/probit/count models		Yes		
ISAT of linear regression		Yes	Yes	Yes
User-specified GETS				Yes
User-specified ISAT				Yes
User-specified diagnostics				Yes
User-specified goodness-of-fit				Yes
Menu-based GUI		Yes	Yes	
Free and open source	Yes		Yes	Yes

Table 1: A comparison of publicly available GETS and ISAT softwares with emphasis on userspecification capabilities. HP1999, the MATLAB code of Hoover and Perez (1999). Autometrics, OxMetrics version 15, see Doornik and Hendry (2018). Grocer, version 1.8, see Dubois and Micheaux (2016). gets, version 0.20, see Sucarrat (2019), and Pretis et al. (2018).

The rest of this article contains four sections. In the next section the model selection properties of GETS and ISAT methods are summarised. This is followed by a section that outlines the general principles of how user-specified estimation, user-specified diagnostics and user-specified goodnessof-fit measures are implemented. Next, a section with four illustrations follows. The final section contains a summary.

## Model selection properties of GETS and ISAT methods

It is useful to denote a generic model for observation t as

$$m(y_t, x_t, \boldsymbol{\beta}), \qquad t = 1, 2, \dots, n,$$
 (1)

where  $y_t$  is the dependent variable,  $x_t = (x_{1t}, x_{2t}, ...)'$  is a vector of covariates,  $\boldsymbol{\beta} = (\beta_1, \beta_2, ...)'$  is a vector of parameters to be estimated and n is the sample size. Two examples are the linear regression model and the logit-model:

$$y_t = \beta_1 x_{1t} + \dots + \beta_k x_{kt} + \epsilon_t, \tag{2}$$

$$y_t = \beta_1 x_{1t} + \dots + \beta_k x_{kt} + \epsilon_t,$$

$$Pr(y_t = 1 | x_t) = \frac{1}{1 + \exp(-h_t)} \quad \text{with} \quad h_t = \beta_1 x_{1t} + \dots + \beta_k x_{kt}.$$
(2)

Note that, in a generic model  $m(y_t, x_t, \beta)$ , the dimension  $\beta$  is usually – but not necessarily – equal to the dimension of  $x_t$ . Here, for notational convenience, they will both have dimension k unless otherwise stated.

In (2)–(3), a variable  $x_{jt} \in x_t$  is said to be relevant if  $\beta_i \neq 0$  and irrelevant if  $\beta_i = 0$ . Let  $k_{rel} \geq 0$  and  $k_{\rm irr} \geq 0$  denote the number of relevant and irrelevant variables, respectively, such that  $k_{\rm rel} + k_{\rm irr} = k$ . GETS modelling aims at finding a specification that contains as many relevant variables as possible, and a proportion of irrelevant variables that on average equals the significance level  $\alpha$  chosen by the investigator. Put differently, if  $\hat{k}_{\rm rel}$  and  $\hat{k}_{\rm irr}$  are the retained number of relevant and irrelevant variables in an empirical application, respectively, then GETS modelling aims at satisfying

$$\mathbb{E}(\widehat{k}_{\text{rel}}/k_{\text{rel}}) \to 1 \quad \text{and} \quad \mathbb{E}(\widehat{k}_{\text{irr}}/k_{\text{irr}}) \to \alpha \quad \text{as} \quad n \to \infty,$$
 (4)

when  $k_{\rm rel}, k_{\rm irr} > 0$ . If either  $k_{\rm rel} = 0$  or  $k_{\rm irr} = 0$ , then the targets are modified in natural ways: If  $k_{\rm rel}=0$ , then the target is  $\mathbb{E}(\widehat{k}_{\rm rel})=0$ , and if  $k_{\rm irr}=0$ , then the target is  $\mathbb{E}(\widehat{k}_{\rm irr})=0$ . Sometimes, the irrelevance proportion  $k_{irr}/k_{irr}$  is also referred to as gauge, whereas the relevance proportion  $k_{irr}/k_{irr}$ is also referred to as potency.

In targeting a relevance proportion equal to 1 and an irrelevance proportion equal to  $\alpha$ , GETS modelling combines well-known ingredients: Multi-path backwards elimination, tests on the  $\beta_i$ 's (both single and multiple hypothesis tests), diagnostics tests and fit-measures (e.g. information criteria). Let  $V(\hat{\beta})$  denote the estimated coefficient-covariance. GETS modelling in the package gets can be described as proceeding in three steps:<sup>2</sup>

- 1. Formulate a General Unrestricted Model (GUM), i.e. a starting model, that passes a set of chosen diagnostic tests. A regressor  $x_j$  in the GUM is non-significant if the p-value of a two-sided t-test is lower than the chosen significance level  $\alpha$ , and each non-significant regressor constitutes the starting point of a backwards elimination path. The test-statistics of the t-tests are computed as  $\widehat{\beta}_j/se(\widehat{\beta}_j)$ , where  $se(\widehat{\beta}_j)$  is the square root of the jth. element of the diagonal of  $V(\widehat{\beta})$ .
- 2. Undertake backwards elimination along multiple paths by removing, one-by-one, non-significant regressors as determined by the chosen significance level  $\alpha$ . Each removal is checked for validity against the chosen set of diagnostic tests, and for parsimonious encompassing (i.e. a multiple hypothesis test) against the GUM. These multiple hypothesis tests on subsets of  $\beta$  are implemented as Wald-tests.
- 3. Multi-path backwards elimination can result in multiple terminal models. The last step of GETS modelling consists of selecting, among the terminal models, the specification with the best fit according to a fit-criterion, e.g. the Schwarz (1978) information criterion.

In ISAT methods, the vector  $x_t$  contains at least n-1 indicators in addition to other covariates that are considered. Accordingly, standard estimation methods are infeasible, since the number of variables in  $x_t$  is usually larger than the number of observations n. The solution to this problem provided by ISAT methods is to first organise  $x_t$  into B blocks:  $x_t^{(1)}, \ldots, x_t^{(B)}$ . These blocks need not be mutually exclusive, so a variable or subset of variables can appear in more than one block. Next, GETS modelling is applied to each block, which leads to B final models. Finally, a new round of GETS modelling is undertaken with the union of the retained variables from the B blocks as covariates in a new starting model (i.e. a new GUM). The model selection properties targeted by ISAT methods are the same as those of GETS methods. Note, however, that since the starting model (the GUM) contains at least n-1 regressors, a tiny significance level - e.g.  $\alpha=0.001$  or smaller - is usually recommended in ISAT methods.

# User-specification: General principles

In the current version of the package <code>gets</code>, version 0.20, the functions that admit user-specified estimation are arx, <code>getsm</code>, <code>getsFun</code> and <code>isat.³</code> The user-specification principles are the same in all four. However, if the result (i.e. a list) returned from the user-specified estimator does not have the same structure as that returned from the default estimator ols (part of the <code>gets</code> package), then arx, <code>getsm</code> and <code>isat</code> may not always work as expected. This is particularly the case with respect to their extraction functions (e.g. <code>print</code>, <code>coef</code>, <code>residuals</code> and <code>predict</code>). User-specified diagnostics and goodness-of-fit functions are optional. By default, <code>getsFun</code> and <code>isat</code> do not perform any diagnostics tests, whereas the default in arx and <code>getsm</code> is to test the standardised residuals for autocorrelation and Autoregressive Heteroscedasticity (ARCH). This is implemented via the diagnostics function (part of the <code>gets</code> package). Also by default, all four functions use the Schwarz (1978) information criterion as goodness-of-fit measure, which favours parsimony.

## The getsFun function

The recommended, most flexible and computationally most efficient approach to user-specified GETS modelling is via the getsFun function. Currently, it accepts up to twenty-five arguments. For the purpose of user-specified estimation, user-specified diagnostics and user-specified goodness-of-fit measures, the most important arguments are:

```
getsFun(y, x,
  user.estimator = list(name = "ols"),
  user.diagnostics = NULL,
  gof.function = list(name = "infocrit", method = "sc"),
  gof.method = c("min", "max"),
  ...)
```

The y is the left-hand side variable (the regressand), x is the regressor or design matrix, user.estimator controls which estimator or model to use and further arguments – if any – to be passed on to the

<sup>&</sup>lt;sup>2</sup>The way GETS modelling is implemented across softwares varies. For example, in Autometrics and Grocer the diagnostics are not checked at each deletion.

<sup>&</sup>lt;sup>3</sup>In the future, the plan is to also enable user-specified GETS modelling with the function getsv, which implements GETS modelling of the log-variance.

estimator, user.diagnostics controls the user-specified diagnostics if any, and gof.function and gof.method control the goodness-of-fit measure used. Note that y and x should satisfy 'is.vector(y) == TRUE' and 'is.matrix(x) == TRUE', respectively, and enter in "clean" ways: If either y or x are objects of class, say, "ts" or "zoo", then getsFun may not behave as expected. By default, the estimator ols is used with its default arguments, which implements OLS estimation via the qr function. The value NULL on user.diagnostics means no diagnostics checks are undertaken by default. The following code illustrates getsFun in linear regression (the default), and reproduces the information printed while searching:

```
n <- 40 #number of observations
k <- 20 #number of Xs

set.seed(123) #for reproducibility
y <- rnorm(n) #generate Y
x <- matrix(rnorm(n*k), n, k) #create matrix of Xs

#do gets w/default estimator (ols), store output in 'result':
result <- getsFun(y, x)

#the information printed while searching:
18 path(s) to search
Searching: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18</pre>
```

The object named result is a list, and the code summary(results) returns a summary of its contents. The most important entries are:

• paths: A list of vectors containing the searched paths. Each vector (i.e. path) indicates the sequence of delection of the regressors. In the example above the first path is

```
$paths[[1]]
[1] 1 15 6 7 3 14 11 16 4 2 8 12 5 9 20 19 13
```

That is, regressor no. 1 was the first to be deleted, regressor no. 15 was the second, regressor no. 6 was the third, and so on.

• terminals: A list of vectors with the distinct terminal models of the specification search. In the example above it is equal to

```
$terminals
$terminals[[1]]
[1] 10 17 18

$terminals[[2]]
[1] 10 18
```

That is, two terminal models. The first contains regressors 10, 17 and 18, whereas the second contains regressors 10 and 18.

• terminals.results: A data frame with the goodness-of-fit information of the terminal models. In the above example the entry is equal to:

spec 1 is short for specification 1, i.e. terminal model 1, and spec 2 is short for specification 2, i.e. terminal model 2. info(sc) indicates that the Schwarz (1978) criterion (the default) is used as goodness-of-fit measure, whereas n and k denote the number of observations and parameters, respectively.

- best.terminal: An integer that indicates which terminal model is the best according to the goodness-of-fit criterion used. In the example above the value is 1.
- specific.spec: A vector of integers that indicates which regressors that are contained in the best terminal model. In the above example it is

```
$specific.spec
[1] 10 17 18
```

That is, the best terminal model contains regressors no. 10, 17 and 18.

#### **User-specified estimation**

User-specified estimation is carried out via the user.estimator argument. It must be a list containing at least one entry – a character – named name with the name of the estimator to be invoked. Optionally, the list can also contain an item named envir, a character, which indicates the environment in which the user-specified estimator resides. If unspecified, then the function is looked for in the global environment (.GlobalEnv). Additional entries in the list, if any, are passed on to the estimator as arguments.

The user-specified estimator must also satisfy the following:

- 1. It should be of the form myEstimator(y,x,...), where y is a vector and x is a matrix. In other words, while the name of the function is arbitrary, the first argument should be the regressand and the second the matrix of covariates.
- 2. The user-defined estimator should return a list with a minimum of six items:
  - n (the number of observations)
  - k (the number of coefficients)
  - df (degrees of freedom, used in the *t*-tests)
  - coefficients (a vector with the coefficient estimates)
  - vcov (the coefficient covariance matrix)
  - logl (a goodness-of-fit value, e.g. the log-likelihood)

The items need not appear in this order. However, the naming should be exactly as indicated. If also the diagnostics and/or the goodness-of-fit criterion is user-specified, then additional objects may be required, see the subsections below on user-specified diagnostics and goodness-of-fit critera. Note also that, if the goodness-of-fit criterion is user-specified, then log1 can in certain situations be replaced by another item (which needs not be named log1).

3. The user-defined estimator must be able to handle NULL regressor-matrices, i.e. situations where either NCOL(x) is 0 or is.null(x) is TRUE. This is needed in situations where a terminal model is empty (i.e. no regressors are retained).

To illustrate how the requirements above can be met in practice, suppose – as an example – that we would like to use the function 1m for estimation rather than ols. The first step is then to make a function that calls 1m while satisfying requirements 1 to 3:

```
lmFun \leftarrow function(y, x, ...)
  ##create list:
  result <- list()
  ##n, k and df:
  result$n <- length(y)
  if( is.null(x) || NCOL(x) == 0){
    result$k <- 0
  }else{
    resultk \leftarrow NCOL(x)
  }
  result$df <- result$n - result$k
  ##call lm if k > 0:
  if( resultk > 0){
    tmp <- lm(y \sim x - 1)
    result$coefficients <- coef(tmp)</pre>
    result$vcov <- vcov(tmp)</pre>
    result$log1 <- as.numeric(logLik(tmp))</pre>
  }else{
    result$coefficients <- NULL
    result$vcov <- NULL
    result$logl <- sum(dnorm(y, sd = sqrt(var(y)), log = TRUE))</pre>
  }
  ##return result:
  return(result)
}
```

The code

```
getsFun(y, x, user.estimator = list(name = "lmFun"))
```

undertakes the same specification search as earlier, but uses 1mFun rather than ols.

## User-specified diagnostics

User-specified diagnostics is carried out via the user.diagnostics argument. The argument must be a list containing at least two entries: A character named name containing the name of the diagnostics function to be called, and an entry named pval that contains a vector with values between 0 and 1, i.e. the chosen significance level(s) for the diagnostics test(s). If only a single test is undertaken by the diagnostics function, then pval should be of length one. If two tests are undertaken, then pval should be of length two. And so on. An example of the argument when only a single test is undertaken is:

```
user.diagnostics = list(name = "myDiagnostics", pval = 0.05))
```

That is, the name of the function is myDiagnostics, and the chosen significance level for the single test that is carried out is 5%. Optionally, just as when the estimator is user-specified, the list can contain an item named envir, a character, which indicates the environment in which the user-specified diagnostics function resides. If unspecified, then also here is the function looked for in the global environment (.GlobalEnv). Also here is it the case that additional items in the list, if any, are passed on to the user-specified function as arguments.

The user-specified diagnostics function must satisfy the following:

- 1. It should be of the form myDiagnostics(result,...), where result is the list returned from the estimator in question, e.g. that of the user-specified estimator (recall requirement 2 in the previous section above).
- 2. It should return an  $m \times 3$  matrix that contains the p-value(s) of the test(s) in the third column, where  $m \ge 1$  is the number of tests carried out. So if only a single test is carried out, then m = 1 and the p-value should be contained in the third column. An example could look like:

```
statistic df pval normality NA NA 0.0734
```

Note that the row-names and column-names in the example are not required. However, they do indicate what kind of information you may wish to put there for reporting purposes, e.g. by using the function diagnostics (also part of the gets package).

To illustrate how the requirements can be met in practice, suppose we would like to ensure the residuals are normal by testing for non-normality with the Shapiro-Wilks test function shapiro.test. In this context, its main argument is the residuals of the estimated model. The list returned by the user-defined estimator named lmFun above, however, does not contain an item with the residuals. The first step, therefore, is to modify the estimator lmFun so that the returned list also contains the residuals:

```
lmFun \leftarrow function(y, x, ...)
  ##info needed for estimation:
  result <- list()
  result$n <- length(y)</pre>
  if( is.null(x) || NCOL(x)==0 ){
    result$k <- 0
  }else{
    resultk \leftarrow NCOL(x)
  result$df <- result$n - result$k
  if( resultk > 0){
    tmp <- lm(y \sim x - 1)
    result$coefficients <- coef(tmp)</pre>
    result$vcov <- vcov(tmp)</pre>
    result$log1 <- as.numeric(logLik(tmp))</pre>
  }else{
    result$coefficients <- NULL
    result$vcov <- NULL
    result$logl <- sum(dnorm(y, sd=sqrt(var(y)), log=TRUE))</pre>
```

```
##residuals:
if( result$k > 0){
   result$residuals <- residuals(tmp)
}else{
   result$residuals <- y
}
return(result)
}</pre>
```

Computationally, the only modification appears under ##residuals. We can now make the user-specified diagnostics function:

```
myDiagnostics <- function(x, ...){
  tmp <- shapiro.test(x$residuals) #do the test
  result <- rbind( c(tmp$statistic, NA, tmp$p.value) )
  return(result)
}</pre>
```

The following code undertakes GETS modelling with the user-specified estimator defined above, and the user-specified diagnostics function using a 5% significance level for the latter:

```
getsFun(y, x, user.estimator = list(name = "lmFun"),
  user.diagnostics = list(name = "myDiagnostics", pval = 0.05))
```

Note that if the chosen significance level for the diagnostics is suffuciently high, then no specification search is undertaken because the starting model does not pass the non-normality test. With the current data, for example, a little bit of trial and error reveals this is the case for a level of about pval = 0.35.

## User-specified goodness-of-fit

User-specified goodness-of-fit is carried out with the gof. function and gof.method arguments. The former indicates which Goodness-of-Fit (GOF) function to use, and the second is a character that indicates whether the best model maximises ("max") or minimises ("min") the GOF criterion in question. The first argument is a list with a structure similar to earlier: It must contain at least one entry, a character named name, with the name of the GOF function to call. An example is:

```
gof.function = list(name = "myGof"))
```

Optionally, also here the list can contain an item named envir, a character, which indicates the environment in which the user-specified GOF function resides, and also here is the user-specified function looked for in the global environment if envir is unspecified. Also as earlier, additional items in the list are passed on to the user-specified GOF function as arguments. The default value, for example, gof.function = list(name = "infocrit",method = "sc"), means the argument method = "sc" is passed on to the function infocrit. The user-specified GOF function must satisfy the following:

- 1. It should be of the form myGof(result,...), where result is the list returned from the estimator in question, e.g. that of the user-specified estimator.
- 2. It should return a single numeric value, i.e. the value of the GOF measure in question.

To illustrate how the requirements can be met in practice, suppose we would like to use the adjusted  $R^2$  as our GOF measure in combination with our user-defined estimator. For the moment, the user-defined estimator 1mFun does not contain the information necessary to compute the adjusted  $R^2$ . In particular, it lacks the regressand y. However, this is readily added:

```
lmFun <- function(y, x, ...){

##info needed for estimation:
  result <- list()
  result$n <- length(y)
  if( is.null(x) || NCOL(x)==0 ){
    result$k <- 0
  }else{
    result$k <- NCOL(x)
  }</pre>
```

```
result$df <- result$n - result$k
if( resultk > 0){
  tmp <- lm(y \sim x - 1)
  result$coefficients <- coef(tmp)</pre>
 result$vcov <- vcov(tmp)</pre>
  result$logl <- as.numeric(logLik(tmp))</pre>
}else{
  result$coefficients <- NULL
  result$vcov <- NULL
 result$logl <- sum(dnorm(y, sd=sqrt(var(y)), log=TRUE))</pre>
##residuals:
if( resultk > 0){
 result$residuals <- residuals(tmp)</pre>
}else{
 result$residuals <- y
}
##info needed for r-squared:
result$y <- y
return(result)
```

The added part appears under ##info needed for r-squared. A GOF function that returns the adjusted  $R^2$  is:

```
myGof <- function(object, ...){
  TSS <- sum((object$y - mean(object$y))^2)
  RSS <- sum(object$residuals^2)
  Rsquared <- 1 - RSS/TSS
  result <- 1 - (1 - Rsquared) * (object$n - 1)/(object$n - object$k)
  return(result)
}</pre>
```

The following code undertakes GETS modelling with all the three user-specified functions defined so far:

```
getsFun(y, x, user.estimator = list(name = "lmFun"),
  user.diagnostics = list(name = "myDiagnostics", pval = 0.05),
  gof.function = list(name = "myGof"), gof.method = "max")
```

Incidentally, it leads to the same final model as when the default GOF function is used.

#### More speed: turbo, max.paths, parallel computing

In multi-path backwards elimination search, one may frequently arrive at a specification that has already been estimated and tested. As an example, consider the following two paths:

```
$paths[[1]]
[1] 2 4 3 1 5

$paths[[2]]
[1] 4 2 3 1 5
```

In path 1, i.e. paths[[1]], regressor no. 2 is the first to be deleted, regressor no. 4 is the second, and so on. In path 2 regressor no. 4 is the first to be deleted, regressor no. 2 is the second, and so on. In other words, after the deletion of the first two variables, the set of remaining variables (i.e. 3, 1 and 5) in the two paths is identical. Accordingly, knowing the result from the first path, in path 2 it is unnecessary to proceed further after having deleted the first two regressors. Setting the argument turbo equal to TRUE turns such a check on, and thus skips searches, estimations and tests that are unnecessary. The turbo comes at a small computational cost, since the check is undertaken at each deletion. This is why the default is turbo = FALSE. However, if the estimation time is noticeable, then turning the turbo on can reduce the search time substantially.

Searching more paths may increase the relevance proportion or potency. Whether and to what extent this happens depends on the sample size n, and on the degree of multicolinearity among the

regressors  $x_t$ . If n is sufficiently large, or if the regressors are sufficiently uncorrelated, then searching fewer paths will not reduce the relevance proportion. In many situations, therefore, one may consider reducing the number of paths to increase the speed. This is achieved via the max.paths argument. Setting max.paths = 10, for example, means a maximum of 10 paths is searched. The paths that are searched are those of the 10 most insignificant variables (i.e. those with the highest p-values) in the starting model.

When implementing ISAT methods, the function isat undertakes a multi-round form of GETS modelling. In the first round the variables are split into *B* blocks, and then GETS modelling is undertaken on each block. This is a socalled "embarassingly parallel" problem. To make isat search in parallel during the first round, simply set the argument parallel.options equal to an integer greater than 1. The integer determines how many cores/threads to use, and the command detectCores() can be used to find out how many cores/threads that are available on the current machine. Remember, it is not recommended to use all the cores/threads available. Within isat, parallel-computing is implemented with the makeCluster and parApply functions from the package **parallel**.

## User-specified GETS and ISAT methods: Illustrations

#### **GETS modelling of Generalised Linear Models (GLMs)**

The function glm enables the estimation of a large number of specifications within the class of Generalised Linear Models (GLMs). Here, it is illustrated how GETS modelling can be implemented with GLMs. To fix ideas, the illustration is in terms of the logit-model.

Let  $y_t \in \{0,1\}$  denote the regressand of the logit-model given by

$$Pr(y_t = 1|x_t) = \frac{1}{1 + \exp(-h_t)}, \qquad h_t = \beta' x_t.$$
 (5)

Next, consider the following set of data:

```
n <- 40 #number of observations
k <- 20 #number of Xs
set.seed(123) #for reproducibility
y <- round(runif(40)) #generate Y
x <- matrix(rnorm(n*k), n, k) #create matrix of Xs</pre>
```

In other words, one regressand  $y_t \in \{0,1\}$  which is entirely independent of the 20 regressors in  $x_t$ . The following function enables GETS modelling of logit-models:

```
logitFun <- function(y, x, ...){</pre>
  ##create list:
  result <- list()
  ##n, k and df:
  result$n <- length(y)
  if( is.null(x) || NCOL(x)==0 ){
    result$k <- 0
  }else{
    result$k <- NCOL(x)
  result$df <- result$n - result$k
  ##call glm if k > 0:
  if( resultk > 0){
    tmp \leftarrow glm(y \sim x - 1, family = binomial(link="logit"))
    result$coefficients <- coef(tmp)</pre>
    result$vcov <- vcov(tmp)</pre>
    result$log1 <- as.numeric(logLik(tmp))</pre>
    result$coefficients <- NULL
    result$vcov <- NULL
    result$log1 <- result$n*log(0.5)
  }
```

```
##return result:
  return(result)
}
```

To undertake the GETS modelling:

```
getsFun(y, x, user.estimator=list(name="logitFun"))
```

Two variables are retained, namely  $x_{5t}$  and  $x_{11t}$ , at the default significance level of 5%. To reduce the chance of retaining irrelavant variables, the significance level can be lowered to, say, 1% by setting t.pval = 0.01.

To implement GETS modelling for a different GLM model, only two lines of code needs to be modified in the user-defined function above. The first is the line that specifies the family, and the second is the one that contains the log-likelihood associated with the empty model (i.e. the line 'result\$logl <-result\$n\*log( $\emptyset$ .5)').

## Creating a gets method (S3) for the "lm" class of models

The package **gets** provides the generic function gets. This enables the creation of GETS methods (S3) for models of arbitrary classes. Here, this is illustrated for models of class "lm". Our aim is to be able to do the following:

```
mymodel <- lm(y ~ x)
gets(mymodel)</pre>
```

That is, to first estimate a model of class "1m", and then to conveniently undertake GETS modelling by simply applying the code gets(.) to the named object mymodel. To this end, a function named gets.1m that relies on getsFun will be created. In doing so, a practical challenge is how to appropriately deal with the intercept. Indeed, as we will see, a notable part of the code in the user-defined function will be devoted to the intercept. Another practical aspect is whether to use 1m or o1s whenever a model is estimated by OLS (both employ the QR decomposition). The latter is simpler codewise and computationally faster, so we opt for the latter. The function is:

```
gets.lm <- function(object, ...){</pre>
  ##make y:
  y <- as.vector(object$model[, 1])</pre>
  yName <- names(object$model)[1]</pre>
  ##make x:
  x <- as.matrix(object$model[, -1])</pre>
  xNames <- colnames(x)</pre>
  if(NCOL(x) == 0){
    x <- NULL
    xNames <- NULL
  }else{
    if(is.null(xNames)){
      xNames <- paste0("X", 1:NCOL(x))</pre>
      colnames(x) <- xNames
    }
  }
  ##is there an intercept?:
  if(length(coef(object)) > 0){
    cTRUE <- names(coef(object))[1] == "(Intercept)"</pre>
    if(cTRUE){
      x \leftarrow cbind(rep(1, NROW(y)), x)
      xNames <- c("(Intercept)", xNames)</pre>
      colnames(x) <- xNames</pre>
    }
  }
  ##do gets:
  myspecific <- getsFun(y, x, ...)</pre>
  ##which are the retained regressors?:
```

```
retainedXs <- xNames[myspecific$specific.spec]
cat("Retained regressors:\n ", retainedXs, "\n")
##return result
return(myspecific)
}</pre>
```

We can now do GETS modelling on models of class "1m" by simply applying the code 'gets(...)' on the object in question. The following code first stores the estimated model of class "1m" in an object named startmodel, and then applies the newly defined function gets.1m to it:

```
startmodel <- lm(y ~ x)
finallm <- gets(startmodel)</pre>
```

The information from the specification search in stored in the object called finallm, and during the search the following is printed:

```
18 path(s) to search
Searching: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
Retained regressors:
X2 X11 X17
```

In other words, the retained regressors are no. 2, 11 and 17.

#### Regression with ARMA error

The function arima can be used to estimate a linear regression with deterministic regressors and an error-term that follows an ARMA. An example is

$$y_t = \beta' x_t + \epsilon_t, \qquad \epsilon_t = \phi_1 \epsilon_{t-1} + \theta_1 u_{t-1} + u_t, \qquad u_t \sim WN(0, \sigma_u^2),$$

where  $x_t$  is a vector of deterministic regressors and WN is short for White Noise. The error  $\epsilon_t$  is thus governed by an ARMA(1,1). Let  $x_t$  denote a (deterministic) step-shift variable in which the step-shift occurs at observation 30, i.e.  $x_t = 1 (t \ge 30)$ . Next, consider the Data Generation Process (DGP) given by

$$y_t = 4x_t + \epsilon_t$$
,  $\epsilon_t = 0.4\epsilon_{t-1} + 0.1u_{t-1} + u_t$ ,  $u_t \sim N(0,1)$ ,  $t = 1, ..., n$ 

with n = 60. In other words, the series  $y_t$  is non-stationary and characterised by a large location shift at t = 30. Figure 1 illustrates the evolution of  $y_t$ , which is obtained with the following code:

```
set.seed(123) #for reproducibility
eps <- arima.sim(list(ar = 0.4, ma = 0.1), 60) #epsilon
x <- coredata(sim(eps, which.ones = 30)) #step-dummy at t = 30
y <- 4*x + eps #the dgp
plot(y, col = "blue", lwd = 2)</pre>
```

By just looking at the graph, it seems clear that there is a location shift, but it is not so clear that it in fact occurs at t=30. I now illustrate how the arima function can be used in combination with getsFun to automatically search for where the break occurs. The idea is to do GETS modelling over a set of step-indicators that cover the period in which the break visually appears to be in. Specifically, the aim is to apply GETS modelling to the following starting model with 11 regressors:

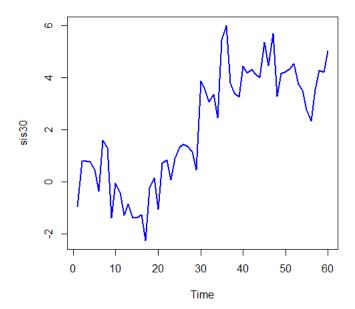
$$y_t = \sum_{i=1}^{11} \beta_i \cdot 1_{\{t \geq 24+i\}} + \epsilon_t, \qquad \epsilon_t = \phi_1 \epsilon_{t-1} + \theta_1 u_{t-1} + u_t.$$

To this end, we first need to make the user-specified estimator:

```
myEstimator <- function(y, x){

##create list:
  result <- list()

##estimate model:
  if( is.null(x) || NCOL(x)==0 ){
    result$k <- 0
    tmp <- arima(y, order = c(1,0,1)) #empty model
}else{</pre>
```



**Figure 1:** The graph of  $y_t$ .

```
result$k <- NCOL(x)
  tmp <- arima(y, order = c(1,0,1), xreg = x)
  result$coefficients <- tmp$coef[-c(1:3)]
  result$vcov <- tmp$var.coef
  result$vcov <- result$vcov[-c(1:3),-c(1:3)]
}

##rename and re-organise things:
  result$n <- tmp$nobs
  result$df <- result$n - result$k
  result$logl <- tmp$loglik

return(result)
}</pre>
```

Note that the estimator has been put together such that the ARMA(1,1) specification of the error  $\epsilon_t$  is fixed. As a consequence, the specification search is only over the regressors. The following code first creates the 11 step dummies, and then undertakes the GETS modelling:

```
xregs <- coredata(sim(eps, which.ones = 25:35)) #11 step-dummies
getsFun(y, xregs, user.estimator = list(name = "myEstimator"))</pre>
```

Two step-dummies are retained, namely those of t = 30 and t = 35.

#### Faster ISAT when n is large

ISAT methods are computationally intensive, since at least n-1 indicators are included as regressors. Accordingly, as n grows large, purpose-specific estimators can greatly reduce the computing time. One way of building such an estimator is by using tools from the package Matrix, see Bates and Maechler (2018). The code below illustrates this. First it loads the library, and then it creates a function named olsFaster that re-produces the structure of the estimation result returned by the function ols with method = 3, but with functions from Matrix. The code is:

```
library(Matrix)
olsFaster <- function(y, x){
  out <- list()
  out$n <- length(y)
  if (is.null(x)){ out$k <- 0 }else{ out$k <- NCOL(x) }</pre>
```

```
out$df <- out$n - out$k
    if (out\$k > 0) {
      x <- as(x, "dgeMatrix")</pre>
      out$xpy <- crossprod(x, y)
      out$xtx <- crossprod(x)
      out$coefficients <- as.numeric(solve(out$xtx,out$xpy))</pre>
      out$xtxinv <- solve(out$xtx)</pre>
      out$fit <- out$fit <- as.vector(x %*% out$coefficients)</pre>
  }else{ out$fit <- rep(0, out$n)</pre>
                                           }
  out$residuals <- y - out$fit
  out$residuals2 <- out$residuals^2
  out$rss <- sum(out$residuals2)</pre>
  out$sigma2 <- out$rss/out$df
  if (out$k > 0) { out$vcov <- as.matrix(out$sigma2 * out$xtxinv) }</pre>
  out logl <- -out n * log(2 * out sigma2 * pi)/2 - out sigma2)
  return(out)
}
```

Depending on the data and hardware/software configuration, the estimator may lead to a considerably speed-improvement. In the following example, the function system.time suggests a speed improvement of about 20% on the current hardware/software configuration:

```
set.seed(123) #for reproducibility
y <- rnorm(1000)
x <- matrix(rnorm(length(y)*20), length(y), 20)
#w/ols:
system.time( finalmodel <- isat(y, mxreg = x, max.paths = 5))
#w/olsFaster:
system.time( finalmodel <- isat(y, mxreg = x, max.paths = 5,
    user.estimator = list(name = "olsFaster")) )</pre>
```

# Summary

In many applications a specific model or estimator is needed, a specific set of diagnostics tests may be required, or a specific fit criterion is preferred. In these situations, the implementation of user-specified GETS and ISAT methods puts a large programming-burden on the user if the combination of estimator/model, diagnostics tests and fit criterion is not already offered by publicly available software. This article has outlined how recent additions to the package gets greatly simplifies the development of user-specified GETS and ISAT methods. The package is the first software – both inside and outside the R universe – to provide a complete set of facilities for user-specified GETS and ISAT methods.

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# **Bibliography**

- D. Bates and M. Maechler. *Matrix: Sparse and Dense Matrix Classes and Methods*, 2018. URL https://CRAN.R-project.org/package=Matrix. R package version 1.2-15. [p12]
- J. Campos, D. F. Hendry, and N. R. Ericsson, editors. General-to-Specific Modeling. Volumes 1 and 2. Edward Elgar Publishing, Cheltenham, 2005. [p1]
- J. Castle, J. Doornik, D. F. Hendry, and F. Pretis. Detecting Location Shifts During Model Selection by Step-Indicator Saturation. *Econometrics*, 3:240–264, 2015. DOI 10.3390/econometrics3020240. [p1]
- J. A. Doornik and D. F. Hendry. *Empirical Econometric Modelling PcGive 15*. Timberlake Consultants Ltd., London, 2018. [p2]

- E. Dubois and E. Micheaux. Grocer 1.72: an econometric toolbox for Scilab. http://dubois.ensae.net/grocer.html, 2016. [p2]
- D. F. Hendry, S. Johansen, and C. Santos. Automatic selection of indicators in a fully saturated regression. *Computational Statistics*, 23:317–335, 2008. DOI 10.1007/s00180-007-0054-z. [p1]
- K. D. Hoover and S. J. Perez. Data Mining Reconsidered: Encompassing and the General-to-Specific Approach to Specification Search. *Econometrics Journal*, 2:167–191, 1999. Dataset and code: http://www.csus.edu/indiv/p/perezs/Data/data.htm. [p1, 2]
- M. C. Lovell. Data Mining. The Review of Economics and Statistics, 65:1–12, 1983. [p1]
- F. Pretis, J. Reade, and G. Sucarrat. Automated General-to-Specific (GETS) Regression Modeling and Indicator Saturation for Outliers and Structural Breaks. *Journal of Statistical Software*, 86:1–44, 2018. [p1, 2]
- G. Schwarz. Estimating the Dimension of a Model. The Annals of Statistics, 6:461–464, 1978. [p3, 4]
- G. Sucarrat. gets: General-to-Specific (GETS) Modelling and Indicator Saturation (ISAT) Methods, 2019. URL https://CRAN.R-project.org/package=gets. R package version 0.20. https://CRAN.R-project.org/package=gets. [p2]

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