MACROECONOMETRÍA

Segundo Cuatrimestre (curso 2007/08), Depto. de Economía, UC3M

Excercise set 5

Question 1. Consider the model

$$\mathbf{y}_t = \mathbf{c} + \mathbf{B_1}\mathbf{y}_{t-1} + \epsilon_t, \qquad \epsilon_t \sim \mathbf{WN}(\mathbf{0}, \Sigma^2), \tag{1}$$

where

$$\mathbf{y}_t = \begin{bmatrix} x_t \\ z_t \end{bmatrix}, \quad \mathbf{c}_t = \begin{bmatrix} 5/6 \\ 6/7 \end{bmatrix}, \quad \mathbf{B_1} = \begin{bmatrix} b & 1/4 \\ 1/2 & 3/4 \end{bmatrix},$$

$$\epsilon_t = \begin{bmatrix} \epsilon_{x,t} \\ \epsilon_{z,t} \end{bmatrix}, \quad \mathbf{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ and } \Sigma^2 = \begin{bmatrix} 3/8 & 0 \\ 0 & 9/10 \end{bmatrix}.$$

a) For which values of $b \geq 0$ is (1) stationary?

b) Suppose now that $b = 2/5$. What are the characteristic roots of $\mathbf{B_1}$. Is (1) stationary?

c) What are the unconditional means of x_t and z_t?

d) Suppose $Var(z_t) = 13.023$ and $Cov(y_t, z_t) = 5.374$. What are the values of $Var(y_t)$ and $Corr(y_t, z_t)$?

Question 2. Consider the model

$$\mathbf{y}_t = \mathbf{c} + \mathbf{B_1}\mathbf{y}_{t-1} + \epsilon_t, \qquad \epsilon_t \sim \mathbf{WN}(\mathbf{0}, \Sigma^2), \tag{2}$$

where

$$\mathbf{y}_t = \begin{bmatrix} x_t \\ y_t \\ z_t \end{bmatrix}, \quad \mathbf{c}_t = \begin{bmatrix} 1/2 \\ 1/3 \\ 1/4 \end{bmatrix}, \quad \mathbf{B_1} = \begin{bmatrix} 1/4 & 2/3 & 1/2 \\ 2/5 & 1/7 & 1/5 \\ 1/6 & 1/3 & 2/7 \end{bmatrix}.$$

a) The characteristic roots of $\mathbf{B_1}$ are 0.954, -0.296 and 0.020. Is (2) stable? What are the solutions of $|\mathbf{I} - \mathbf{B_1}L| = 0$?

b) What are the unconditional means of x_t, y_t and z_t?

c) Write (2) in companion form.

Question 3. Consider the structural model

$$\mathbf{y}_t = \mathbf{B_0}\mathbf{y}_t + \mathbf{B_1}\mathbf{y}_{t-1} + \mathbf{e}_t, \tag{3}$$

where

$$\mathbf{y}_t = \begin{bmatrix} x_t \\ z_t \end{bmatrix}, \ \mathbf{B_0} = \begin{bmatrix} 0 & b_{12.0} \\ b_{21.0} & 0 \end{bmatrix} \text{ and } \mathbf{B_1} = \begin{bmatrix} b_{11.1} & b_{12.1} \\ b_{21.1} & b_{22.1} \end{bmatrix},$$

and where $\mathbf{e}_t$ is an error vector.

a) What is the reduced form of (3)?

b) The reduced form of (3) can be written as

$$\mathbf{y}_t = \mathbf{D_1}\mathbf{y}_{t-1} + \mathbf{u}_t. \tag{4}$$

Suppose we obtain the estimates

$$\hat{\mathbf{D}}_\mathbf{1} = \begin{bmatrix} 1/3 & 0 \\ 1/3 & 2/3 \end{bmatrix} \tag{5}$$

by means of OLS estimation. Considering these estimates only (that is, not caring about inference), do the estimates suggest that z_t Granger-causes x_t and vice-versa?

c) Given the estimates $\hat{\mathbf{D}}_\mathbf{1}$ the two parameters in $\mathbf{B_0}$ and the four parameters in $\mathbf{B_1}$ are unidentified unless we impose identifying restrictions. Suppose now that common sense, economic theory, both, or whatever that may inform your beliefs, suggests that $b_{12.0} = 0$, that is, that z_t has no contemporaneous impact on x_t. Are any parameters (in addition to $b_{12.0}$) in $\mathbf{B_0}$ and $\mathbf{B_1}$ identified? If so, what are their values? Does x_t Granger-cause z_t and vice-versa (again, do not care about inference)?

Question 4. Let $\mathbf{y'}_t = [c_t, i_t] \sim I(1)$, say, c_t is log of household consumption and i_t is log of disposable income, and consider the VEqCM

$$\Delta\mathbf{y}_t = \mathbf{B_0}\Delta\mathbf{y}_t + \mathbf{B_1}\Delta\mathbf{y}_{t-1} + \theta\alpha'\mathbf{y}_{t-1} + \epsilon_t, \quad \epsilon_t \sim IID(\mathbf{0}, \Sigma^2) \tag{6}$$

where

$$\mathbf{B_0} = \begin{bmatrix} 0 & b_{12.0} \\ b_{21.0} & 0 \end{bmatrix}, \mathbf{B_1} = \begin{bmatrix} b_{11.1} & b_{12.1} \\ b_{21.1} & b_{22.1} \end{bmatrix}, \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}, \alpha = \begin{bmatrix} 1 \\ \alpha_{21} \end{bmatrix}.$$

a) Write (6) in equation form without vector and matrix notation.

b) Suppose that $\Pi = \theta\alpha' = \begin{bmatrix} \frac{-1}{3} & \frac{-9}{30} \\ \frac{1}{9} & \frac{1}{10} \end{bmatrix}$. What are the characteristic roots of Π? What is the rank of Π? Are c_t and i_t cointegrated?

c) Given Π, are the parameters in θ and α identified? If so, what are their values?

d) Suppose that $\mathbf{B_0} = \begin{bmatrix} 0 & \frac{1}{4} \\ 0 & 0 \end{bmatrix}$ and that $\mathbf{B_1} = \begin{bmatrix} \frac{-1}{3} & \frac{1}{5} \\ 0 & \frac{1}{2} \end{bmatrix}$. What are the Granger-causality relations between c_t and i_t? Are any of the variables weakly or strongly exogenous?

e) Given your conclusions in d), to what extent is a single-equation analysis valid of c_t?

Question 5. Let $\mathbf{y'}_t = [c_t, i_t, w_t] \sim I(1)$, say, c_t is log of household consumption, i_t is log of household income and w_t is log of household wealth, and consider the VEqCM

$$\Delta\mathbf{y}_t = \mathbf{B_0}\Delta\mathbf{y}_t + \mathbf{B_1}\Delta\mathbf{y}_{t-1} + \theta\alpha'\mathbf{y}_{t-1} + \epsilon_t, \quad \epsilon_t \sim IID(\mathbf{0}, \Sigma^2) \tag{7}$$

where

$$\mathbf{B_0} = \begin{bmatrix} 0 & b_{12.0} & b_{13.0} \\ b_{21.0} & 0 & b_{23.0} \\ b_{31.0} & b_{32.0} & 0 \end{bmatrix}, \mathbf{B_1} = \begin{bmatrix} b_{11.1} & b_{12.1} & b_{13.1} \\ b_{21.1} & b_{22.1} & b_{23.1} \\ b_{31.1} & b_{32.1} & b_{33.1} \end{bmatrix}, \theta = \begin{bmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \\ \theta_{31} & \theta_{32} \end{bmatrix}, \alpha = \begin{bmatrix} 1 & 1 \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{bmatrix}.$$

a) Write (7) in equation form without vector and matrix notation.

b) Suppose that $\Pi = \theta\alpha' = \begin{bmatrix} \frac{-8}{15} & \frac{-9}{30} & \frac{-1}{25} \\ 0 & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{20} \end{bmatrix}$. What are the characteristic roots of Π? What is the rank of Π? How many cointegration relations are there?

c) Suppose statistical hypothesis testing suggests that $\alpha_{31} = 0$ and that $\alpha_{22} = 0$. Use this information and the values of Π to identify the parameters in θ and the rest of the parameters in α.

d) Suppose $\mathbf{B_0} = \begin{bmatrix} 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{7} & 0 \end{bmatrix}$ and $\mathbf{B_1} = \begin{bmatrix} \frac{1}{3} & \frac{1}{6} & 0 \\ 0 & \frac{1}{2} & 0 \\ \frac{-1}{10} & \frac{1}{4} & \frac{1}{2} \end{bmatrix}$. Are any of the variables weakly or strongly exogenous?

e) Is a single-equation analysis of c_t justified?