MACROECONOMETRÍA

Segundo Cuatrimestre (curso 2006/07), Depto. de Economía, UC3M

Excercise set 5

Question 1. Consider the model

$$\mathbf{y}_t = \mathbf{c} + \mathbf{B_1}\mathbf{y}_{t-1} + \epsilon_t, \qquad \epsilon_t \sim \mathbf{WN}(\mathbf{0}, \Sigma^2), \tag{1}$$

where

$$\mathbf{y}_t = \begin{bmatrix} x_t \\ z_t \end{bmatrix}, \ \mathbf{c}_t = \begin{bmatrix} 5/6 \\ 6/7 \end{bmatrix}, \ \mathbf{B_1} = \begin{bmatrix} b & 1/4 \\ 1/2 & 3/4 \end{bmatrix},$$

$$\epsilon_t = \begin{bmatrix} \epsilon_{x,t} \\ \epsilon_{z,t} \end{bmatrix}, \ \mathbf{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ and } \Sigma^2 = \begin{bmatrix} 3/8 & 0 \\ 0 & 9/10 \end{bmatrix}.$$

a) For which values of $b \geq 0$ is (1) stationary?

b) Suppose now that $b = 2/5$. What are the characteristic roots of $\mathbf{B_1}$. Is (1) stationary?

c) What are the unconditional means of x_t and z_t?

d) Suppose $Var(z_t) = 13.023$ and $Cov(y_t, z_t) = 5.374$. What are the values of $Var(y_t)$ and $Corr(y_t, z_t)$?

Question 2. Consider the model

$$\mathbf{y}_t = \mathbf{c} + \mathbf{B_1}\mathbf{y}_{t-1} + \epsilon_t, \qquad \epsilon_t \sim \mathbf{WN}(\mathbf{0}, \Sigma^2), \tag{2}$$

where

$$\mathbf{y}_t = \begin{bmatrix} x_t \\ y_t \\ z_t \end{bmatrix}, \ \mathbf{c}_t = \begin{bmatrix} 1/2 \\ 1/3 \\ 1/4 \end{bmatrix}, \ \mathbf{B_1} = \begin{bmatrix} 1/4 & 2/3 & 1/2 \\ 2/5 & 1/7 & 1/5 \\ 1/6 & 1/3 & 2/7 \end{bmatrix}.$$

a) The characteristic roots of $\mathbf{B_1}$ are 0.954, -0.296 and 0.020. Is (2) stable? What are the solutions of $|\mathbf{I} - \mathbf{B_1}L| = 0$?

b) What are the unconditional means of x_t, y_t and z_t?

c) Write (2) in companion form.

Question 3. Consider the structural model

$$\mathbf{y}_t = \mathbf{B_0}\mathbf{y}_t + \mathbf{B_1}\mathbf{y}_{t-1} + \mathbf{e}_t, \tag{3}$$

where

$$\mathbf{y}_t = \begin{bmatrix} x_t \\ z_t \end{bmatrix}, \quad \mathbf{B_0} = \begin{bmatrix} 0 & b_{12.0} \\ b_{21.0} & 0 \end{bmatrix} \text{ and } \mathbf{B_1} = \begin{bmatrix} b_{11.1} & b_{12.1} \\ b_{21.1} & b_{22.1} \end{bmatrix},$$

and where $\mathbf{e}_t$ is an error vector.

a) What is the reduced form of (3)?

b) The reduced form of (3) can be written as

$$\mathbf{y}_t = \mathbf{D_1}\mathbf{y}_{t-1} + \mathbf{u}_t. \tag{4}$$

Suppose we obtain the estimates

$$\hat{\mathbf{D}}_\mathbf{1} = \begin{bmatrix} 1/3 & 0 \\ 1/3 & 2/3 \end{bmatrix} \tag{5}$$

by means of OLS estimation. Considering these estimates only (that is, not caring about inference), do the estimates suggest that z_t Granger-causes x_t and vice-versa?

c) Given the estimates $\hat{\mathbf{D}}_\mathbf{1}$ the two parameters in $\mathbf{B_0}$ and the four parameters in $\mathbf{B_1}$ are unidentified unless we impose identifying restrictions. Suppose now that common sense, economic theory, both, or whatever that may inform your beliefs, suggests that $b_{12.0} = 0$, that is, that z_t has no contemporaneous impact on x_t. Are any parameters (in addition to $b_{12.0}$) in $\mathbf{B_0}$ and $\mathbf{B_1}$ identified? If so, what are their values? Does x_t Granger-cause z_t and vice-versa (again, do not care about inference)?