MACROECONOMETRÍA

Segundo Cuatrimestre (curso 2006/07), Depto. de Economía, UC3M

Excercise set 6

Question 1. Let $\mathbf{y'}_t = [c_t, i_t] \sim I(1)$, say, c_t is log of household consumption and i_t is log of disposable income, and consider the VEqCM

$$\Delta \mathbf{y}_t = \mathbf{B_0} \Delta \mathbf{y}_t + \mathbf{B_1} \Delta \mathbf{y}_{t-1} + \theta \alpha' \mathbf{y}_{t-1} + \epsilon_t, \quad \epsilon_t \sim IID(\mathbf{0}, \Sigma^2) \tag{1}$$

where

$$\mathbf{B_0} = \begin{bmatrix} 0 & b_{12.0} \\ b_{21.0} & 0 \end{bmatrix}, \mathbf{B_1} = \begin{bmatrix} b_{11.1} & b_{12.1} \\ b_{21.1} & b_{22.1} \end{bmatrix}, \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}, \alpha = \begin{bmatrix} 1 \\ \alpha_{21} \end{bmatrix}.$$

a) Write (1) in equation form without vector and matrix notation.

b) Suppose that $\Pi = \theta\alpha' = \begin{bmatrix} \frac{-1}{3} & \frac{-9}{30} \\ \frac{1}{9} & \frac{1}{10} \end{bmatrix}$. What are the characteristic roots of Π? What is the rank of Π? Are c_t and i_t cointegrated?

c) Given Π, are the parameters in θ and α identified? If so, what are their values?

d) Suppose that $\mathbf{B_0} = \begin{bmatrix} 0 & \frac{1}{4} \\ 0 & 0 \end{bmatrix}$ and that $\mathbf{B_1} = \begin{bmatrix} \frac{-1}{3} & \frac{1}{5} \\ 0 & \frac{1}{2} \end{bmatrix}$. What are the Granger-causality relations between c_t and i_t? Are any of the variables weakly or strongly exogenous?

e) Given your conclusions in *d*), to what extent is a single-equation analysis valid of c_t?

Question 2. Let $\mathbf{y'}_t = [c_t, i_t, w_t] \sim I(1)$, say, c_t is log of household consumption, i_t is log of household income and w_t is log of household wealth, and consider the VEqCM

$$\Delta \mathbf{y}_t = \mathbf{B_0} \Delta \mathbf{y}_t + \mathbf{B_1} \Delta \mathbf{y}_{t-1} + \theta \alpha' \mathbf{y}_{t-1} + \epsilon_t, \quad \epsilon_t \sim IID(\mathbf{0}, \Sigma^2) \tag{2}$$

where

$$\mathbf{B_0} = \begin{bmatrix} 0 & b_{12.0} & b_{13.0} \\ b_{21.0} & 0 & b_{23.0} \\ b_{31.0} & b_{32.0} & 0 \end{bmatrix}, \mathbf{B_1} = \begin{bmatrix} b_{11.1} & b_{12.1} & b_{13.1} \\ b_{21.1} & b_{22.1} & b_{23.1} \\ b_{31.1} & b_{32.1} & b_{33.1} \end{bmatrix}, \theta = \begin{bmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \\ \theta_{31} & \theta_{32} \end{bmatrix}, \alpha = \begin{bmatrix} 1 & 1 \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{bmatrix}.$$

a) Write (2) in equation form without vector and matrix notation.

b) Suppose that $\Pi = \theta\alpha' = \begin{bmatrix} \frac{-8}{15} & \frac{-9}{30} & \frac{-1}{25} \\ 0 & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{20} \end{bmatrix}$. What are the characteristic roots of Π? What is the rank of Π? How many cointegration relations are there?

c) Suppose statistical hypothesis testing suggests that $\alpha_{31} = 0$ and that $\alpha_{22} = 0$. Use this information and the values of Π to identify the parameters in θ and the rest of the parameters in α.

d) Suppose $\mathbf{B_0} = \begin{bmatrix} 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{7} & 0 \end{bmatrix}$ and $\mathbf{B_1} = \begin{bmatrix} \frac{1}{3} & \frac{1}{6} & 0 \\ 0 & \frac{1}{2} & 0 \\ \frac{-1}{10} & \frac{1}{4} & \frac{1}{2} \end{bmatrix}$. Are any of the variables weakly or strongly exogenous?

d) Is a single-equation analysis of c_t justified?